

Changes of State and Mixtures

How much energy to melt, to boil, and where does it all end up?

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A mocktail, some ice, and a number

(slide 2)

Here is a question from the bar, and I would like you to keep it open until the end of the lesson. Why do bartenders put so much ice in cocktails? To give you less drink, or is there a physical reason? Let us see what physics says, with a mocktail, that is a non-alcoholic cocktail, so that alcohol does not come into play. It is 20 cl, about 0.20 kg, at room temperature: 27°C , that is 300 K. The ice comes straight from the freezer, at -18°C , in cubes of about 20 g each. We want the ice to cool the drink *without all melting*, or the drink gets watered down. How many cubes do we need so that some of them survive? Ignore the glass and the room.

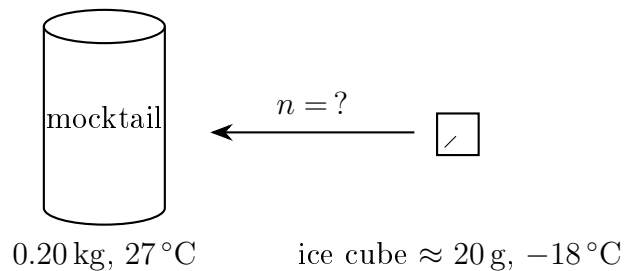


Figure 1: The question: how many cubes? Schematic, not to scale.

Before going on
Make a guess before we do any physics: one or two cubes, three, four, or eight or more? Write your number down. We will come back to it.

To answer, we need two things we do not know well enough yet: how much energy it costs to melt ice, and what happens to the temperature when two things at different temperatures are put together. Let us start with the first.

Heating at constant power

(slide 3)

Take 1 kg of ice at -20°C and heat it with a heater of constant power, say 1000 W, so that no energy is lost to the surroundings. Every second the heater delivers 1000 J, which means that one kilojoule of energy is one second of heating. Figure 2 shows the temperature as a function of the energy supplied, drawn to scale.

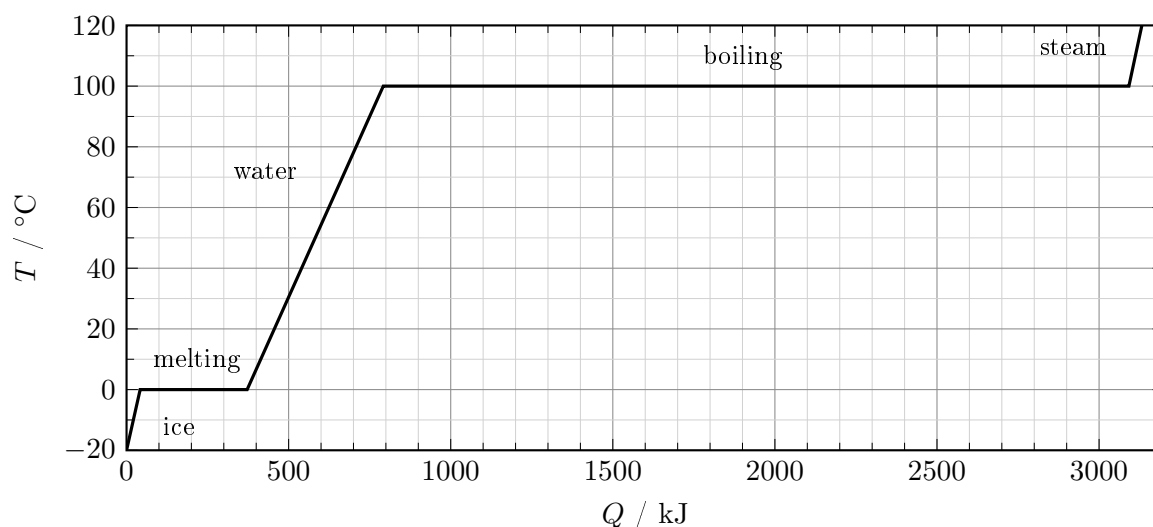


Figure 2: Temperature of 1 kg of ice heated at constant power, as a function of the energy supplied. Drawn to scale.

Let us read it in order. At first the ice warms, from -20°C to 0°C . Then the line goes flat: the temperature stays at 0°C while energy keeps flowing in, and ice and water are present together. When all the ice has melted, the water warms from 0°C to 100°C . Then comes another plateau, much longer: the water boils at 100°C . Only when it has all become steam does the temperature rise again.

Before going on

During a plateau the temperature does not change: the thermometer stays still. But the heater is still on, delivering 1000 J every second. The energy keeps going in and the temperature does not rise: where does it go?

Remember that temperature is tied to the average kinetic energy of the particles. If the temperature does not change, the average kinetic energy of the particles does not change either. So the energy is not making the particles move faster. It goes into the arrangement of the particles: it is used to overcome the attractions that hold them in a rigid solid, or that keep them together in a liquid. That energy was supplied, but the thermometer does not show it. This is why it is called *latent*, hidden, heat.

Because the drawing is to scale, two things can be read from it. First, the slopes: the line for ice is about twice as steep as the line for water, and so is the line for steam. The same energy raises the temperature of ice twice as much as that of water, because ice needs about half the energy per kilogram per degree: $2100 \text{ J}/(\text{kg } ^{\circ}\text{C})$ against $4200 \text{ J}/(\text{kg } ^{\circ}\text{C})$. Second, the lengths: the melting plateau takes about a tenth of the whole horizontal axis, and the boiling plateau almost three quarters of it.

Step	What happens to 1 kg	Energy
1	ice warms from $-20\text{ }^{\circ}\text{C}$ to $0\text{ }^{\circ}\text{C}$	42 kJ
2	ice melts at $0\text{ }^{\circ}\text{C}$	330 kJ
3	water warms from $0\text{ }^{\circ}\text{C}$ to $100\text{ }^{\circ}\text{C}$	420 kJ
4	water boils at $100\text{ }^{\circ}\text{C}$	2300 kJ
5	steam warms from $100\text{ }^{\circ}\text{C}$ to $120\text{ }^{\circ}\text{C}$	40 kJ
total		3132 kJ

One question decides the formula

(slide 4)

We now have two different situations, and two different formulas. When the temperature is changing, the energy goes into the motion of the particles, and

$$Q = m c \Delta T,$$

where c is the specific heat capacity, in $\text{J}/(\text{kg } ^{\circ}\text{C})$, which is different for ice, water and steam, and $\Delta T = T_{\text{final}} - T_{\text{initial}}$. When the state is changing at constant temperature, the energy goes into the arrangement of the particles, and

$$Q = m L,$$

where L is the specific latent heat, in J/kg : L_f for melting and freezing (fusion), L_v for boiling and condensing (vaporisation).

The question to ask before writing any term
Is the temperature changing, or is the state changing?

Quantitative calorimetry of this kind goes beyond the 0653 syllabus: it is an extension.

How big are the latent heats?

(slide 5)

In this lesson we use rounded values: $c_{\text{ice}} = 2100 \text{ J}/(\text{kg } ^\circ\text{C})$, $c_{\text{water}} = 4200 \text{ J}/(\text{kg } ^\circ\text{C})$, $c_{\text{steam}} = 2000 \text{ J}/(\text{kg } ^\circ\text{C})$, $L_f = 3.3 \times 10^5 \text{ J/kg}$, $L_v = 2.3 \times 10^6 \text{ J/kg}$.

Is melting cheaper or more expensive than warming water by one hundred degrees? Melting 1 kg of ice costs 330 kJ, and warming 1 kg of water from 0°C to 100°C costs 420 kJ: they are of the same order. Melting costs as much as warming the same water by about 79°C . Boiling is a different story. It costs 2300 kJ, about 5.5 times the energy needed to warm the same water from 0°C to 100°C , and about 7 times the energy needed to melt it. So the larger latent heat is the one of vaporisation, and by far. In melting, the particles only become free to slide past one another; in boiling they have to be separated completely.

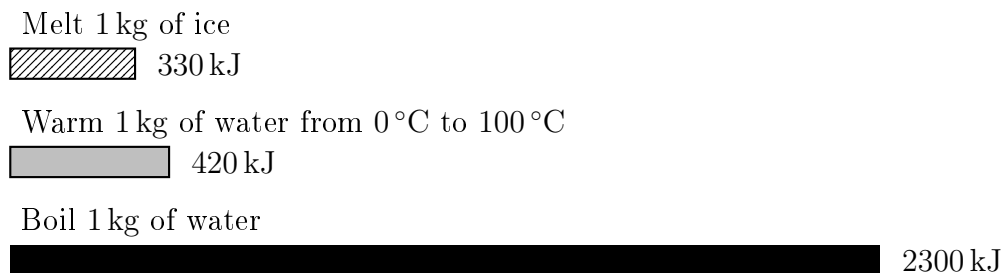


Figure 3: Energy needed for 1 kg: melting, warming water from 0°C to 100°C , boiling. Drawn to scale.

Worked example

(slide 6)

How much energy is needed to turn 200 g of ice at -10°C into water at 30°C ? The method is the same each time: one step per line, and for each step we ask whether the temperature is changing or the state is changing. Always $\Delta T = T_{\text{final}} - T_{\text{initial}}$, and masses in kilograms.

$Q_1 = 0.20 \times 2100 \times 10 = 4200 \text{ J}$	the ice warms from -10°C to 0°C
$Q_2 = 0.20 \times 3.3 \times 10^5 = 66\,000 \text{ J}$	the ice melts at 0°C
$Q_3 = 0.20 \times 4200 \times 30 = 25\,200 \text{ J}$	the water warms from 0°C to 30°C

$$Q = Q_1 + Q_2 + Q_3 = 95\,400 \text{ J} \approx 95 \text{ kJ}.$$

Melting is by far the largest term.

Your turn: one step at a time

(slide 7)

1. How much energy heats 250 g of water from 20°C to 70°C ?
2. How much energy melts 0.50 kg of ice that is already at 0°C ?
3. How much energy evaporates 100 g of water that is already at 100°C ?

Your turn: time, and a burn

(slide 8)

4. A 1000 W heater warms 0.50 kg of ice, starting from -20°C , and loses no energy. How long does it take until: (a) the ice reaches 0°C ; (b) all of it has melted; (c) the water reaches 100°C ; (d) all of it has boiled away? Sketch the temperature against time.

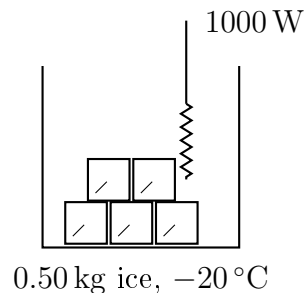


Figure 4: Exercise 4: ice heated by a 1000 W heater. Schematic.

5. 10 g of steam at 100°C , or 10 g of boiling water at 100°C , lands on your skin and cools to 34°C . Compare the energy released in the two cases.

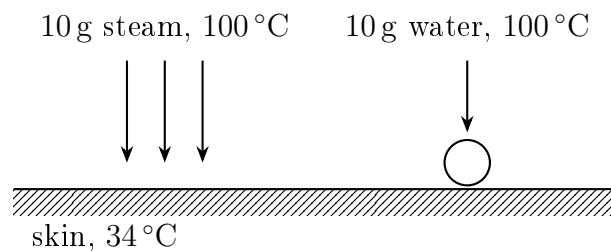


Figure 5: Exercise 5: steam or boiling water on the skin. Schematic.

Mixtures: one loses, one gains

(slides 9–10)

Now put two bodies at different temperatures in contact, and let nothing escape to the outside. All the heat that leaves one body enters the other:

$$Q_{\text{lost}} = Q_{\text{gained}}.$$

Let us see how this sentence translates, starting from the body that warms up. Body 1 is the colder one (mass m_1 , specific heat capacity c_1 , initial temperature T_1); body 2 is the hotter one (m_2, c_2, T_2). The two bodies end at the same temperature, the *equilibrium temperature* T_{eq} , with $T_1 < T_{\text{eq}} < T_2$: it is the unknown. We call it T_{eq} , and not “final temperature”, so that it is never confused with the “final” of $\Delta T = T_{\text{final}} - T_{\text{initial}}$.

The cold body warms up and gains energy. Its temperature rises from T_1 to T_{eq} , and the energy gained is, exactly as in every formula so far, final minus initial:

$$Q_{\text{gained}} = m_1 c_1 (T_{\text{eq}} - T_1).$$

The hot body cools down and loses energy. Its temperature drops from T_2 to T_{eq} . Final minus initial would give $T_{\text{eq}} - T_2$, which is negative; but the energy lost is a positive quantity, so we flip it:

$$Q_{\text{lost}} = m_2 c_2 (T_2 - T_{\text{eq}}).$$

Notice that here T_{eq} comes second: it is not a mistake, it is the flip. T_{eq} is the same in the two brackets, because it is the temperature at which both bodies end. Let us put the two expressions into the equality:

$$m_1 c_1 (T_{\text{eq}} - T_1) = m_2 c_2 (T_2 - T_{\text{eq}}).$$

This equation is already correct and is enough to solve the problem. But it has a drawback: the two bodies are written with different rules, “final minus initial” for one and “initial minus final” for the other, and with three bodies it becomes easy to make mistakes. So let us use *one rule for everybody*, always final minus initial, with the equilibrium temperature as the final temperature of every body:

$$Q = m c (T_{\text{eq}} - T_{\text{initial}}).$$

For the cold body nothing changes: $Q_1 = m_1 c_1 (T_{\text{eq}} - T_1)$ is positive and is exactly Q_{gained} . For the hot body, instead,

$$Q_2 = m_2 c_2 (T_{\text{eq}} - T_2) = -m_2 c_2 (T_2 - T_{\text{eq}}) = -Q_{\text{lost}}.$$

This is where the minus sign comes from. With the rule “final minus initial”, $T_{\text{eq}} - T_2$ is negative because the body cools down, so Q_2 is negative: it is the energy lost, seen from the body that lost it, and it is the opposite of the positive quantity Q_{lost} . We did not add the minus sign: it is already inside $(T_{\text{eq}} - T_2)$. Let us substitute $Q_{\text{lost}} = -Q_2$ into $Q_{\text{lost}} = Q_{\text{gained}}$:

$$-Q_2 = Q_1 \implies Q_1 + Q_2 = 0 \quad \left(\sum Q = 0 \right).$$

Moving $-Q_2$ to the other side of the equals sign changes the sign again, and we are left with $Q_1 + Q_2 = 0$: the minus of Q_2 , which is negative, and the minus due to changing side have cancelled. The result is a single equation, with no absolute values and no need to decide in advance who loses and who gains, because the signs tell us by themselves. It works just as well with three bodies: $Q_1 + Q_2 + Q_3 = 0$.

Worked example: hot water into cold water

(slide 11)

0.20 kg of water at 80 °C is poured into 0.30 kg of water at 20 °C. What is the equilibrium temperature? Before any algebra we can say where it will be: between 20 °C and 80 °C, and closer to 20 °C, because there is more cold water. Both bodies end at the same T_{eq} . The cold water is body 1, the hot water is body 2.

First way of writing it, with lost and gained, each one positive:

$$\underbrace{0.20 \cdot 4200 (80 - T_{\text{eq}})}_{Q_{\text{lost}}} = \underbrace{0.30 \cdot 4200 (T_{\text{eq}} - 20)}_{Q_{\text{gained}}}.$$

Second way, with the single rule “final minus initial” and $\sum Q = 0$, the cold body first:

$$\underbrace{0.30 \cdot 4200 (T_{\text{eq}} - 20)}_{Q_1 > 0} + \underbrace{0.20 \cdot 4200 (T_{\text{eq}} - 80)}_{Q_2 < 0} = 0.$$

They are the same equation: the second is obtained from the first by moving the lost term to the other side of the equals sign and noting that $-(80 - T_{\text{eq}}) = T_{\text{eq}} - 80$. The specific heat capacity is the same and cancels:

$$0.30 (T_{\text{eq}} - 20) + 0.20 (T_{\text{eq}} - 80) = 0 \Rightarrow 0.50 T_{\text{eq}} = 22 \Rightarrow T_{\text{eq}} = 44^\circ\text{C}.$$

It is between 20 °C and 80 °C, closer to 20 °C: the check works. With the numbers: $Q_{\text{lost}} = 0.20 \cdot 4200 \cdot 36 = 30\,240 \text{ J}$ and $Q_{\text{gained}} = 0.30 \cdot 4200 \cdot 24 = 30\,240 \text{ J}$.

When ice is involved: budget first

(slide 12)

If one of the bodies is ice, there is a trap in the equation: we do not know in advance whether the ice melts completely, only partly, or not at all, and the melting term must be written only if it has happened. So before any equation we do the *budget*: how much energy can the hot body give by cooling down to 0 °C? How much does the ice need to warm up to 0 °C, and then to melt completely?

There are three possible endings. If the hot body can pay for everything, all the ice melts, $T_{\text{eq}} > 0^\circ\text{C}$, and $\sum Q = 0$ contains a melting term. If it can pay for warming the ice but not for melting all of it, only part of the ice melts and the mixture stays at $T_{\text{eq}} = 0^\circ\text{C}$, with some ice surviving. If it cannot even bring the ice to 0 °C, no ice melts, $T_{\text{eq}} < 0^\circ\text{C}$ and there is no melting term. Only then do we write $\sum Q = 0$, for that ending only, and solve. At the end we check that T_{eq} is inside the range that ending allows.

The mocktail and the ice: the budget in action

(slide 13)

Let us try it with the mocktail: 0.20 kg at 27 °C, and four cubes, that is 80 g of ice at -18°C . By cooling to 0°C the mocktail can give $0.20 \times 4200 \times 27 = 22\,680\text{ J}$. The ice needs $0.080 \times 2100 \times 18 = 3024\text{ J}$ to warm up to 0°C , and $0.080 \times 3.3 \times 10^5 = 26\,400\text{ J}$ to melt completely: 29 424 J in all, more than the mocktail can give. So not all the ice melts. After paying for the warming, $22680 - 3024 = 19\,656\text{ J}$ are left for melting, so the mass melted is $19656/3.3 \times 10^5 \approx 0.060\text{ kg}$: about 60 g melt and about 20 g survive. The equilibrium temperature is $T_{\text{eq}} = 0^{\circ}\text{C}$, and the meltwater is about $60/(200 + 60) \approx 23\%$ of the drink.

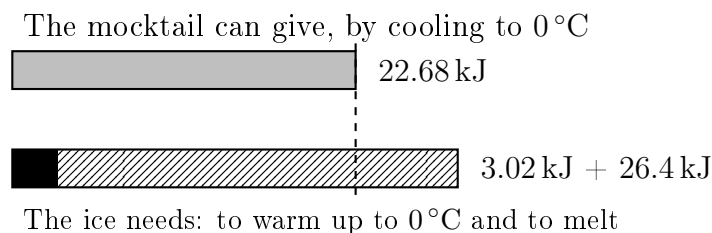


Figure 6: The budget for the mocktail with four cubes (80 g) of ice at -18°C . Drawn to scale; dashed line: what the mocktail can give.

Before going on

The same mocktail, but now with eight cubes, that is 160 g of ice at -18°C . What is the equilibrium temperature? How much ice survives, and how much water is added to the drink?

The budget answers. The mocktail can always give 22 680 J. The ice needs $0.16 \times 2100 \times 18 = 6048\text{ J}$ just to warm up, which leaves $22680 - 6048 = 16\,632\text{ J}$ for melting. Melting all the ice would need $0.16 \times 3.3 \times 10^5 = 52\,800\text{ J}$, far more. So the ice melts only partly, $T_{\text{eq}} = 0^{\circ}\text{C}$, and the mass melted is $16632/3.3 \times 10^5 \approx 0.050\text{ kg}$: about 50 g melt and about 110 g survive. The meltwater is $50/(200 + 50) = 20\%$ of the drink: with twice the cubes the drink is watered down a little *less*, because there is more ice to warm up from -18°C to 0°C , and less energy is left for melting.

Your turn: mixtures

(slide 14)

6. A block of metal of mass 0.30 kg at 100 °C is dropped into 0.50 kg of water at 20 °C. The equilibrium temperature is 25 °C. Find the specific heat capacity of the metal. Which metal might it be?

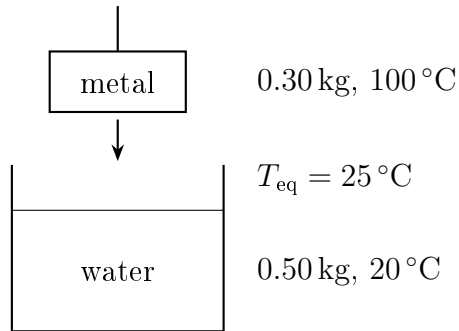


Figure 7: Exercise 6: a hot metal block dropped into water. Schematic.

7. 0.20 kg of tea at 80 °C. Compare adding 50 g of ice at 0 °C with adding 50 g of water at 0 °C: what is the equilibrium temperature in each case?

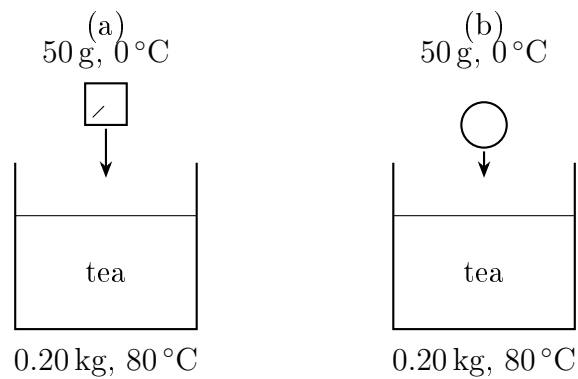


Figure 8: Exercise 7: (a) ice added to the tea; (b) cold water added to the tea. Schematic.

Your turn: does it all melt? Steam as a heater

(slide 15)

8. 0.50 kg of water at 20°C is poured onto 0.40 kg of ice at 0°C in an insulated container. What is in the container at the end, and at what temperature?

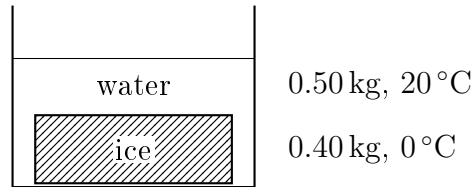


Figure 9: Exercise 8: water on ice in an insulated container. Schematic.

9. A coffee machine blows 20 g of steam at 100°C into 200 g of milk at 5°C . Treat the milk as water. What is the equilibrium temperature?

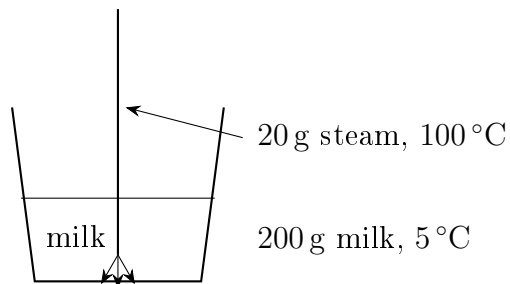


Figure 10: Exercise 9: steam blown into milk. Schematic.

The one from Reels: lava on ice

(slides 16–17)

One kilogram of lava at 1100°C lands on a block of ice of mass 100 kg at -10°C . Assume that the contact is perfect and that no energy escapes. Take the specific heat capacity of lava as about $1000\text{ J}/(\text{kg }^{\circ}\text{C})$ and ignore its solidification.

Before going on

When nothing changes any more, how much of the ice has melted: all of it, some of it, or none at all?

10. Find the equilibrium temperature and the mass of ice that melts. Start with the budget.

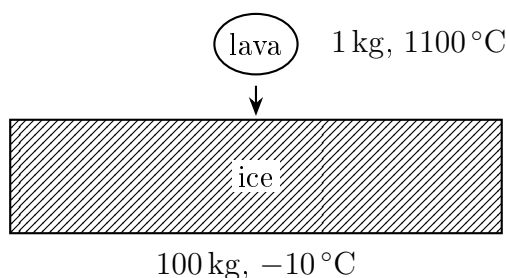


Figure 11: Exercise 10: lava on a block of ice. Schematic.

The budget gives the answer before any equation. By cooling down to 0°C the lava can give $1 \times 1000 \times 1100 = 1100\text{ kJ}$. The ice needs $100 \times 2100 \times 10 = 2100\text{ kJ}$ just to warm up to 0°C : more than the lava can give. So no ice melts at all, and in the ideal case everything ends below 0°C , at about -4.7°C . Melting all of the ice would need another $33\,000\text{ kJ}$. The point is the order of magnitude: a very hot but light body against a cold and heavy one. Real lava is not like this, because contact is not perfect and the lava also releases energy when it solidifies; the ideal model tells us what the energy budget allows.

The lava can give, by cooling to 0°C

1100 kJ

The ice needs, just to warm up to 0°C

2100 kJ

Figure 12: The budget of the lava and the ice (exercise 10). Drawn to scale.

Back to the mocktail

(slides 18–19)

At the beginning we asked why bartenders use so much ice, and how many cubes are needed for some of them to survive. We now have all the tools. By cooling to 0°C the mocktail can give

$$Q_{\text{drink}} = 0.20 \times 4200 \times 27 = 22\,680 \text{ J}.$$

Every kilogram of ice, to melt completely, has to warm up to 0°C and then melt:

$$2100 \times 18 + 3.3 \times 10^5 = 367\,800 \text{ J/kg}.$$

So the mass of ice that the mocktail can melt completely is

$$m = \frac{22680}{367800} \approx 0.062 \text{ kg} \approx 62 \text{ g},$$

which is about 3.1 cubes of 20 g. With three cubes (60 g) all the ice melts and the drink ends at about 0.6°C; with four cubes (80 g) the energy is not enough to melt them all: about 20 g survive. The answer is four.

There is a more interesting result than the number, though. Beyond four cubes the drink still gives only 22 680 J, enough to melt about 50–60 g of ice whatever the number of cubes. The extra cubes do not end up in the glass: they stay ice and keep the drink at 0°C for as long as they last. With eight cubes, 110 g of ice survive and the meltwater is 20% of the drink, less than with four. This is the answer to the opening question: a bartender who uses a lot of ice is not giving you more water, he is giving you a colder drink, for longer.

The model has limits, and it is fair to say so. It describes how cold and how watery the drink is at equilibrium, not how fast: big cubes melt more slowly because they have less surface. A real cocktail contains alcohol, which lowers the freezing point: that is why we started from a mocktail.

In summary

(slide 20)

When the temperature is changing we use $Q = mc\Delta T$; when the state is changing at constant temperature we use $Q = mL$. Latent heats are huge: melting 1 kg of ice costs as much as warming water by 79°C, and boiling costs 5.5 times as much as warming the same water from 0°C to 100°C. For mixtures we write $\sum Q = 0$, with $Q = mc(T_{\text{eq}} - T_i)$ for every body. And whenever ice is involved, we do the budget first, the equation second, and the check last.

Solutions

1. $Q = mc \Delta T = 0.25 \times 4200 \times (70 - 20) = 1050 \times 50 = 52\,500 \text{ J} = 52.5 \text{ kJ}.$
2. $Q = mL_f = 0.50 \times 3.3 \times 10^5 = 165\,000 \text{ J} = 165 \text{ kJ}.$
3. $Q = mL_v = 0.10 \times 2.3 \times 10^6 = 230\,000 \text{ J} = 230 \text{ kJ}.$ This is more than the energy of exercise 2, for one fifth of the mass.
4. Each step takes the energy divided by the power, $t = Q/P.$
 - (a) $Q = 0.50 \times 2100 \times 20 = 21\,000 \text{ J},$ so $t = 21 \text{ s}.$
 - (b) $Q = 0.50 \times 3.3 \times 10^5 = 165\,000 \text{ J},$ so this step takes 165 s: total $21 + 165 = 186 \text{ s}.$
 - (c) $Q = 0.50 \times 4200 \times 100 = 210\,000 \text{ J},$ so this step takes 210 s: total $186 + 210 = 396 \text{ s}.$
 - (d) $Q = 0.50 \times 2.3 \times 10^6 = 1\,150\,000 \text{ J},$ so this step takes 1150 s: total $396 + 1150 = 1546 \text{ s},$ about 26 min.

The graph is the curve of Figure 2 with time on the horizontal axis; the plateaus last 165 s and 1150 s.

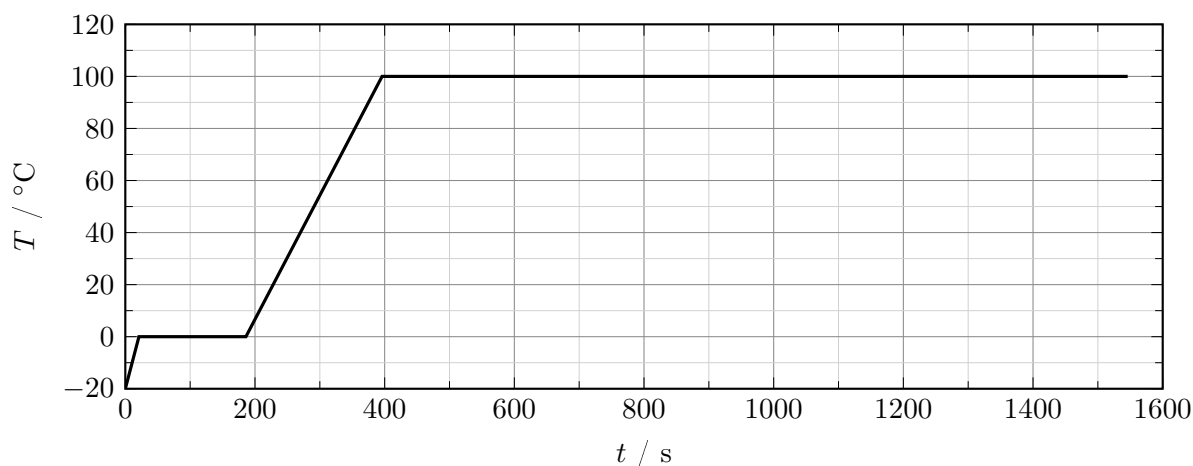


Figure 13: Solution of exercise 4: temperature against time. Drawn to scale.

5. Steam: condensing releases $0.010 \times 2.3 \times 10^6 = 23\,000 \text{ J},$ then the water cools from 100°C to $34^\circ\text{C},$ releasing $0.010 \times 4200 \times 66 = 2772 \text{ J}.$ Total $25\,772 \text{ J}.$ Boiling water: only $2772 \text{ J}.$ The ratio is $25772/2772 \approx 9.3:$ the steam releases about nine times as much energy.

6. $\sum Q = 0$ with $Q = mc(T_{\text{eq}} - T_i):$

$$0.30c(25 - 100) + 0.50 \times 4200 \times (25 - 20) = 0 \Rightarrow -22.5c + 10500 = 0.$$

So $c = 10500/22.5 \approx 467 \text{ J}/(\text{kg } ^\circ\text{C}),$ about $4.7 \times 10^2 \text{ J}/(\text{kg } ^\circ\text{C}):$ compatible with iron or steel (about 450–500). We have assumed that no energy is lost to the container or the air.

7. (a) With water at $0^\circ\text{C}:$ $0.20 \times 4200 \times (T_{\text{eq}} - 80) + 0.050 \times 4200 \times (T_{\text{eq}} - 0) = 0,$ so $0.25T_{\text{eq}} = 16$ and $T_{\text{eq}} = 64^\circ\text{C}.$

(b) With ice: budget first. The tea can give, by cooling to 0°C , $0.20 \times 4200 \times 80 = 67\,200\text{ J}$; melting the ice needs $0.050 \times 3.3 \times 10^5 = 16\,500\text{ J}$. The tea can pay for it, so all the ice melts:

$$0.20 \cdot 4200 (T_{\text{eq}} - 80) + 0.050 \cdot 3.3 \times 10^5 + 0.050 \cdot 4200 T_{\text{eq}} = 0$$

gives $840 T_{\text{eq}} - 67200 + 16500 + 210 T_{\text{eq}} = 0$, so $1050 T_{\text{eq}} = 50700$ and $T_{\text{eq}} \approx 48.3^\circ\text{C}$. The ice cools the tea much more than the same mass of cold water, 48.3°C against 64°C : the difference is the latent heat.

8. Budget: the water can give, by cooling to 0°C , $0.50 \times 4200 \times 20 = 42\,000\text{ J}$. Melting all the ice would need $0.40 \times 3.3 \times 10^5 = 132\,000\text{ J}$, much more. So only part of the ice melts and $T_{\text{eq}} = 0^\circ\text{C}$. The mass melted is $42000/3.3 \times 10^5 \approx 0.127\text{ kg}$. At the end the container holds about 0.273 kg of ice and $0.50 + 0.127 = 0.627\text{ kg}$ of water, at 0°C .

9. The steam condenses, releasing mL_v , so its term in $\sum Q = 0$ is negative; then the water formed cools from 100°C to T_{eq} ; the milk warms from 5°C to T_{eq} :

$$-0.020 \times 2.3 \times 10^6 + 0.020 \times 4200 (T_{\text{eq}} - 100) + 0.20 \times 4200 (T_{\text{eq}} - 5) = 0,$$

that is $-46000 + 84 T_{\text{eq}} - 8400 + 840 T_{\text{eq}} - 4200 = 0$, so $924 T_{\text{eq}} = 58600$ and $T_{\text{eq}} \approx 63.4^\circ\text{C}$. Check: it lies between 5°C and 100°C , so all the steam did condense. Twenty grams of steam warm the milk by about 58°C : the latent heat of vaporisation at work.

10. Budget: the lava can give $1 \times 1000 \times 1100 = 1.1 \times 10^6\text{ J}$; the ice needs $100 \times 2100 \times 10 = 2.1 \times 10^6\text{ J}$ just to reach 0°C . It is not enough, so no ice melts and there is no melting term:

$$1 \times 1000 (T_{\text{eq}} - 1100) + 100 \times 2100 (T_{\text{eq}} + 10) = 0$$

gives $1000 T_{\text{eq}} - 1100000 + 210000 T_{\text{eq}} + 2100000 = 0$, so $211000 T_{\text{eq}} = -1000000$ and $T_{\text{eq}} \approx -4.7^\circ\text{C}$. Check: it is below 0°C , consistent with no melting. Melting all the ice would need $100 \times 3.3 \times 10^5 = 3.3 \times 10^7\text{ J}$. Assumptions: perfect contact, no energy lost, lava stays liquid (its solidification is ignored).