

Solutions: heat, power and temperature change, practice sheet

Self-marking: award the marks part by part, as shown. Total: 50 marks. The lifebuoy parts are worth 30 marks (60%): with 30 marks or more you are on the right track. Where a number differs slightly from yours because of rounding, it is still correct.

1.

(a) Given: $P = 800 \text{ W}$, $m = 2.0 \text{ kg}$, $t = 5.0 \text{ min} = 300 \text{ s}$, $\Delta T = 60^\circ\text{C}$. Unknown: c .  [1]

(b) $E = Pt$ and $Q = mc\Delta T$, so $c = \frac{Q}{m\Delta T}$.  [2]

(c) $E = 800 \times 300 = 240\,000 \text{ J}$. [1]

(d) With no loss $Q = E$: $c = \frac{240\,000}{2.0 \times 60} = 2000 \text{ J/(kg}^\circ\text{C)}$. [1]

Check and common mistakes. Minutes not converted: $E = 4000 \text{ J}$ and $c = 33 \text{ J/(kg}^\circ\text{C)}$, far too small for any material. Division by m forgotten: $240\,000/60 = 4000 \text{ J/}^\circ\text{C}$, which is the thermal capacity C , not c . Reasonableness: oil has about half the specific heat capacity of water (2000 against 4200).

2.

(a) Given: $P = 50 \text{ W}$, $t = 10 \text{ min} = 600 \text{ s}$, $m = 0.50 \text{ kg}$, $c = 900 \text{ J/(kg}^\circ\text{C)}$, initial temperature 20°C . Unknown: ΔT without loss, and the efficiency.  [1]

(b) $\Delta T = \frac{Q}{mc}$; efficiency = $\frac{\text{useful energy}}{\text{total energy supplied}} \times 100\%$.  [2]

(c) $E = 50 \times 600 = 30\,000 \text{ J}$; $\Delta T = \frac{30\,000}{0.50 \times 900} = 66.7 \approx 67^\circ\text{C}$. [1]

(d) Measured rise: $74 - 20 = 54^\circ\text{C}$; $Q = 0.50 \times 900 \times 54 = 24\,300 \text{ J}$; efficiency = $\frac{24\,300}{30\,000} \times 100 = 81\%$. [1]

(e) Insulate the block (for example with a layer of foam or fibre) to reduce the energy lost to the surroundings. [1]

Check and common mistakes. Using 74°C as the rise: $Q = 33\,300 \text{ J}$ and efficiency 111% , impossible. Minutes not converted: $E = 500 \text{ J}$ and $\Delta T = 1.1^\circ\text{C}$. Reasonableness: the measured rise (54°C) must be smaller than the rise without loss (67°C), and the efficiency below 100% .

3.

(a) Given: $m = 0.25 \text{ kg}$, $c = 4200 \text{ J/(kg}^\circ\text{C)}$, $\Delta T = 20^\circ\text{C}$, $P = 40 \text{ W}$. Unknown: t .  [1]

(b) $Q = mc\Delta T$ and $E = Pt$; with $E = Q$: $t = \frac{mc\Delta T}{P}$.  [2]

(c) $Q = 0.25 \times 4200 \times 20 = 21\,000 \text{ J}$. [1]

(d) $t = \frac{21\,000}{40} = 525 \text{ s} = \frac{525}{60} = 8.75 \text{ min} \approx 8.8 \text{ min}$. [1]

Check and common mistakes. Answer left in seconds (525) when minutes were asked. $\Delta T = 80^\circ\text{C}$ (the starting temperature) instead of the fall of 20°C : $Q = 84\,000 \text{ J}$ and $t = 35 \text{ min}$. Reasonableness: a cup of hot water loses 20°C in some minutes, not in seconds or hours.

4.

(a) Given: $A = 2.0 \text{ m}^2$, intensity 600 W/m^2 , efficiency 50% , $m = 100 \text{ kg}$, $t = 1.0 \text{ h} = 3600 \text{ s}$, $c = 4200 \text{ J/(kg}^\circ\text{C)}$. Unknown: ΔT . [1]

(b) $P_{\text{useful}} = \text{efficiency} \times \text{intensity} \times A$; $\Delta T = \frac{P_{\text{useful}} t}{mc}$. [2]

(c) Power received: $600 \times 2.0 = 1200 \text{ W}$; useful power $= 0.50 \times 1200 = 600 \text{ W}$. [1]

(d) $E = 600 \times 3600 = 2.16 \times 10^6 \text{ J}$. [1]

(e) $\Delta T = \frac{2.16 \times 10^6}{100 \times 4200} = 5.14 \approx 5.1^\circ\text{C}$. [1]

Check and common mistakes. The 50% forgotten: $\Delta T = 10.3^\circ\text{C}$. Hour not converted ($t = 1.0$): $E = 600 \text{ J}$ and $\Delta T = 0.0014^\circ\text{C}$. Reasonableness: 100 kg of water has a large thermal capacity, so a rise of a few degrees in an hour is plausible.

5.

(a) Given: $P = 8.0 \text{ kW} = 8000 \text{ W}$; in $t = 1 \text{ s}$ the mass of water is $m = 0.10 \text{ kg}$; entering temperature 12°C ; $c = 4200 \text{ J/(kg}^\circ\text{C)}$. Unknown: ΔT . [1]

(b) Energy in 1 s : $E = Pt = 8000 \text{ J}$; $\Delta T = \frac{Q}{mc}$ with $Q = E$. [2]

(c) $\Delta T = \frac{8000}{0.10 \times 4200} = 19.0 \approx 19^\circ\text{C}$. [1]

(d) $12 + 19 = 31^\circ\text{C}$. [1]

Check and common mistakes. Kilowatts used as watts: $\Delta T = 0.019^\circ\text{C}$. The rise (19°C) given as the outlet temperature instead of $12 + 19 = 31^\circ\text{C}$. Reasonableness: a shower at 31°C is warm and plausible.

6.

(a) Given: $\Delta L = 1.9 \text{ mm} = 1.9 \times 10^{-3} \text{ m}$, $\Delta T = 120 - 20 = 100^\circ\text{C}$, $\alpha = 1.9 \times 10^{-5}/^\circ\text{C}$. Unknown: L_0 . [1]

(b) $\Delta L = \alpha L_0 \Delta T$, so $L_0 = \frac{\Delta L}{\alpha \Delta T}$. [2]

(c) $L_0 = \frac{1.9 \times 10^{-3}}{1.9 \times 10^{-5} \times 100} = 1.0 \text{ m}$. [1]

Check and common mistakes. Millimetres not converted: $L_0 = 1000 \text{ m}$. $\Delta T = 120^\circ\text{C}$ instead of 100°C : $L_0 = 0.83 \text{ m}$. Reasonableness: a laboratory bar is about 1 m long, not a kilometre.

7.

(a) Given: $L_0 = 1.50 \text{ m}$, $\Delta L = 2.07 \text{ mm} = 2.07 \times 10^{-3} \text{ m}$, $\alpha = 2.3 \times 10^{-5}/^\circ\text{C}$, initial temperature 18°C . Unknown: ΔT and the final temperature. [1]

(b) $\Delta L = \alpha L_0 \Delta T$, so $\Delta T = \frac{\Delta L}{\alpha L_0}$. [2]

(c) $\Delta T = \frac{2.07 \times 10^{-3}}{2.3 \times 10^{-5} \times 1.50} = 60^\circ\text{C}$. [1]

(d) $18 + 60 = 78^\circ\text{C}$. [1]

Check and common mistakes. L_0 forgotten: $\Delta T = 90^\circ\text{C}$. Millimetres not converted: $\Delta T = 60\,000^\circ\text{C}$, absurd. The rise (60°C) given as the final temperature. Reasonableness: 78°C is a temperature easily reached with hot water.

8.

(a) Given: side 0.50 m , $\Delta T = 220 - 20 = 200^\circ\text{C}$, $\alpha = 1.2 \times 10^{-5}/^\circ\text{C}$. Unknown: ΔA . [1]

(b) $\Delta A = \beta A_0 \Delta T$ and $\beta = 2\alpha$. [2]

(c) $A_0 = 0.50^2 = 0.25 \text{ m}^2$; $\beta = 2.4 \times 10^{-5}/^\circ\text{C}$; $\Delta A = 2.4 \times 10^{-5} \times 0.25 \times 200 = 1.2 \times 10^{-3} \text{ m}^2 = 12 \text{ cm}^2$. [1]

Check and common mistakes. α used instead of β : 6.0 cm^2 . The side (0.50 m) used instead of the area: 24 cm^2 . Wrong conversion of m^2 to cm^2 (factor 100 instead of 10^4): 0.12 cm^2 . Reasonableness: $\Delta A/A_0 = 0.48\%$, a small change.

9.

(a) Given: $m = 1.0 \text{ kg}$, $\Delta T = 65 - 15 = 50^\circ\text{C}$, $t = 4.0 \text{ min} = 240 \text{ s}$, $c = 4200 \text{ J}/(\text{kg}^\circ\text{C})$. Unknown: P . [1]

(b) $Q = mc \Delta T$ and $P = \frac{E}{t}$; with $E = Q$: $P = \frac{mc \Delta T}{t}$. [2]

(c) $Q = 1.0 \times 4200 \times 50 = 210\,000 \text{ J}$; $P = \frac{210\,000}{240} = 875 \text{ W}$. [1]

(d) Electrical power $= \frac{875}{0.80} = 1094 \text{ W} \approx 1.1 \text{ kW}$. [1]

Check and common mistakes. Multiplying by 0.80 instead of dividing: 700 W , less than the useful power, which is impossible. Minutes not converted: $P = 52\,500 \text{ W}$. Reasonableness: about 1 kW , like a small kettle.

10.

(a) Given: $P = 25 \text{ W}$, $t = 8.0 \text{ min} = 480 \text{ s}$, $\Delta T = 12^\circ\text{C}$, $c = 4200 \text{ J}/(\text{kg}^\circ\text{C})$. Unknown: m . [1]

(b) $E = Pt$ and $Q = mc \Delta T$; with $E = Q$: $m = \frac{Pt}{c \Delta T}$. [2]

(c) $E = 25 \times 480 = 12\,000 \text{ J}$. [1]

(d) $m = \frac{12\,000}{4200 \times 12} = 0.238 \approx 0.24 \text{ kg}$. [1]

Check and common mistakes. Minutes not converted: $E = 200 \text{ J}$ and $m = 0.004 \text{ kg}$ (4 g), a spoonful, not a mug. Reasonableness: 0.24 kg is about 240 mL , a mug of tea.